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VIBRATION ANALYSIS AND MULTI-OBJECTIVE OPTIMIZATION OF STIFFENED TRIANGULAR PLATE

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ABSTRACT

In this paper, nonlinear vibration of a triangular shape plate, with several stiffeners, is studied. The governing equation of transversal deflection of the plate, with considering the effects of orthotropic characteristics and external excitation, is analyzed. The ordinary differential equation for the time response of the system, through employing the Galerkin method, is obtained; and the frequency response of the plate-shape structure - using the multiple scale method - is determined. A robust genetic-based multi-objective optimization technique is employed to optimize the system's response by finding the optimum values of the geometry and locations of the plate's stiffeners. The influence of various parameters on the optimization results is investigated. According to the results, the optimum design of the stiffeners leads to a better performance of the vibration response.

INTRODUCTION

Due to comprehensive applications of triangular-shape plates in various industries such as ship structures, ceilings, aircrafts bodies, vehicles, and so on; investigation of their mechanical behavior and optimal design of them are crucial for the researchers and manufacturers. Surveying the literature reveals that, one of the main aspects of their behavior is the vibrational response of such structures, with and without external excitations. Consequently, a number of research works have been dedicated for the vibration analyses; which are mostly based on numerical methods [1]. As examples, the natural frequencies and mode shapes of triangular-shape plates with free vibration were accomplished by Leissa and Jabber using the Ritz method [2]. Considering various boundary conditions, the free vibration of thin triangular plates was investigated by means of Rayleigh-Ritz method [3]. Hocine et al. applied the hp- finite element for vibration analysis of the orthotropic triangular and rectangular-shape plates [4]. Nallima et al. studied a triangular composite plate where its edges elastically restrained [5]. In majority of studies, the linearization theories with considering small amplitudes of vibration have been taken into account.

Optimal design of stiffeners for the plate form structures and investigating the effects of stiffeners on the response of corresponding structures are valuable contributions toward improving their mechanical behavior in variant applications. As previous works, large oscillation analysis of an anisotropic triangular plate including stiffeners was conducted by Nowliski and Ismail [6]; Shastry and Rao investigated the vibrations of thin rectangular plates having arbitrary oriented stiffeners [7]; Wu and Liu] analyzed the vibration of stiffened plates with elastically restrained edges [8]. The vibration characteristics of anisotropic plates -with eccentric stiffeners- and eigenvalue sensitivity analysis of stiffened plates -with respect to the stiffener locations- were investigated in [9-10].

Marcelin used genetic algorithm-based optimization for optimal design of stiffened plates [11]; he utilized shells and stiffened plates with an FE mesh support. Philen and Wang utilized active stiffeners for vibration control of a circular plate [12]. Adeli et al. optimized types of composite plates including piezoelectric stiffener-actuators [13] and the effects of considering single mode approach were studied [14]. Elsabbagh accomplished the size optimization of stiffeners for a bending plate using a finite element based method [15].

The current work investigates the nonlinear vibration of orthotropic triangular plates. Double-rib shape stiffeners are considered for the plate and also it is subjected to external harmonic excitation. The Galerkin method is applied to governing coupled partial differential equations of the plate deflection and stress. Accordingly, a nonlinear ordinary differential equation for the deflection time response is obtained. There exist various perturbative and non-perturbative methods for analyzing of the obtained equation [16-17].

A multiple-scale method is utilized to analyze nonlinear systems and to obtain the frequency response for the plate vibration. The resulted multi-objective optimization problem for the frequency response is solved and the optimum sizes and positions for the stiffeners are obtained. Genetic Algorithm is chosen here as a well-established optimizer to solve our multi-objective problem [18]. This method, which is an evolutionary one, is applicable for solving many complicated problems in mechanical design applications [19]. Finally, simulation results are presented for various numerical dimensions of plate and stiffeners; also, the optimum frequency responses are computed and compared for various scenarios.

MATHEMATICAL MODELING

An elastic right-angle triangular-shape plate having lengths of *A* and *B* in the respective -x and -y directions and a uniform thickness of *h*, shown in Figure 1, is considered. The Von-Karman governing equations of motion are developed as [6]:

$$\Gamma_1(w,\psi) = w_{xxxx} + 2m^2 w_{xxyy} + k^2 w_{yyyy} + \frac{\rho h}{D_1} w_{tt} + \frac{\mu_0}{D_1} w_t - \frac{q_0}{D_1} q(x,y,t) - \frac{h}{D_1} L(w,\psi) = 0 \quad (1)$$

$$\Gamma_2(w,\psi) = \psi_{xxxx} + p^2 \psi_{xxyy} + k^2 \psi_{yyyy} + \frac{E_2}{2} L(w,w) = 0 \quad (2)$$

In these equations, w and ψ indicate the deflection of the plate in the -z direction and the stress function, respectively.

$$\sigma_{xx} = \psi_{yy}, \quad \sigma_{yy} = \psi_{xx}, \quad \sigma_{xy} = \psi_{xy} \quad (3)$$

$$L(w,\psi) = w_{xx}\psi_{yy} - 2w_{xy}\psi_{xy} + w_{yy}\psi_{xx} \quad (4)$$

$$r^2 = \frac{D_2}{D_1}, \quad d^2 = \frac{D_3}{D_1}, \quad z^2 = \frac{E_2}{G_{12}} - 2\upsilon_2 \quad (5)$$

Parameters D_1 and D_2 are the two bending rigidities in the -x and -y directions, and hence

$$\begin{cases} D_1 = H = \dfrac{Eh^3}{12(1-\upsilon_1^2)} \\ D_2 = \dfrac{Eh^3}{12(1-\upsilon_2^2)} + \dfrac{EI}{s} \\ D_3 = (D_1\upsilon_2 + D_2\upsilon_1)/2 + 2D_k\ ; \quad \text{and} \quad D_k = G_{12}h^3/12 \end{cases} \quad (6)$$

where E_1 and υ_1 are the respective Young's elasticity module and Poisson's ratio in the -x direction. E_2 and υ_2 are those in the -y direction, respectively; and G_{12} is the shear modulus of rigidity. Also, in Eq. (1), ρ and μ denote density and damping coefficients; and, q_0 is the amplitude of external excitation. The deflection terms and the stress functions, as a separable form of displacement and time function, are written as

$$w = g(x,y).\tau(t) \quad \text{and} \quad \psi = g(x,y).\phi(t) \quad (7)$$

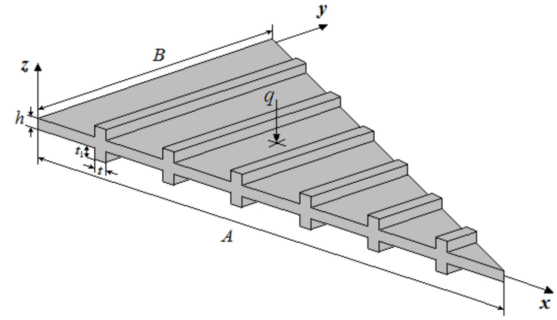

Figure 1: Schematic model of a triangular-shape plate with six stiffeners and excitation load *q*

Considering the fundamental mode of vibration for the plate, in which, the displacement and stress are zero at each contour and maximum at the central portion; the trial shape function is assumed as follow in such a way that all the boundary conditions are satisfied [6]:

$$g(x,y) = x^2 y^2 \left(1 - \frac{x}{a} - \frac{y}{b}\right)^2 \quad (8)$$

Substituting Eq. (7) into Eq. (1) and Eq. (2), the following integrations are constructed, based on the Galerkin method.

$$\int_0^b \int_0^a \Gamma_1(w,\psi) g(x,y)\, dxdy = 0 \quad (9)$$

$$\int_0^b \int_0^a \Gamma_2(w,\psi) g(x,y)\, dxdy = 0 \quad (10)$$

Finding $\theta(t)$ from the second integration and utilizing it in the solution of the first one, the governing equation for the time response of the plate deflection is obtained as

$$\ddot{\tau} + \omega^2\tau + 2\hat{\mu}\dot{\tau} + \alpha\tau^3 = \hat{q}q(t), \quad (11)$$

where

$$\begin{cases} \omega^2 = \dfrac{a_1 + a_2 + a_3}{a_4}, \ 2\hat{\mu} = \dfrac{a_5}{a_4} = 2\varepsilon\mu, \\ \alpha = \dfrac{(a_5 - a_4)(a_8 - a_7)}{(b_1 + b_2 + b_3)a_4}, \ \hat{q} = \dfrac{a_6}{a_4} = \varepsilon q \end{cases} \quad (12)$$

$$\begin{cases} a_1 = \dfrac{334}{525}ab^5, a_2 = \dfrac{5702}{575}m^2a^3b^3, \\ a_3 = \dfrac{334}{575}k^2a^5b, a_4 = \dfrac{257\ \rho h}{27050\ D_1}a^5b^5, \\ a_5 = \dfrac{257\ \mu_0}{27050\ D_1}a^5b^5, a_6 = \dfrac{13q_0}{360\ D_1}a^3b^3, \\ a_7 = \dfrac{8064422\ h}{2910600\ D_1}a^5b^5,\ a_8 = \dfrac{1525214\ h}{415800\ D_1}a^5b^5, \\ b_1 = \dfrac{334}{525}ab^5,\ b_2 = \dfrac{2851}{1575}p^2a^3b^3, b_3 = \dfrac{334}{525}k^2a^5b, \\ b_4 = \dfrac{403221}{2910600}E_2a^5b^5,\ b_5 = \dfrac{762607}{415800}E_2a^5b^5 \end{cases} \quad (13)$$

It should be noted that, for this study the first shape function is considered for the plate vibration. Because this function is the most important and commonly used mode shape in the plate vibration; and it has the lowest natural frequency known as the fundamental natural frequency. On the other hand, considering higher modes increases the number of objectives to more than two; which makes it very complicated to solve and analyze.

PROBLEM SOLUTION

Eq. (11) shows a form of nonlinear equation having hardening cubic nonlinearity and external force, which is taken harmonically as $q(t) = \cos(\Omega t)$. The equation has been solved by the multiple scales method (MMS); which is a well-known perturbation technique [16]. Because of the page number restriction, we do not aim to discuss the details of the solution methodology here as it can be found in corresponding references; and we just present the obtained solution.

Here, a critical case, in which the excitation frequency Ω is close to the natural frequency ω, is considered and consequently primary resonances will happen in the system. Accordingly, one will set $\Omega = \omega + \varepsilon\sigma$, where ε is the non-dimensional small parameter and σ is the detuning parameter. The frequency response of the system - in terms of natural frequency, nonlinear coefficient, damping ratio, and amplitude of excitation - is obtained as [16]:

$$\sigma = \frac{3}{8}\frac{\alpha}{\omega}a^2 \pm \sqrt{\frac{q^2}{4\omega^2 a^2} - \mu^2} \quad , \tag{14}$$

where a denotes the amplitude of vibration; and σ represents the proximity of excitation and natural frequencies. Plotting above response for two typical cases, one can reach the graphs presented in Figure 2.

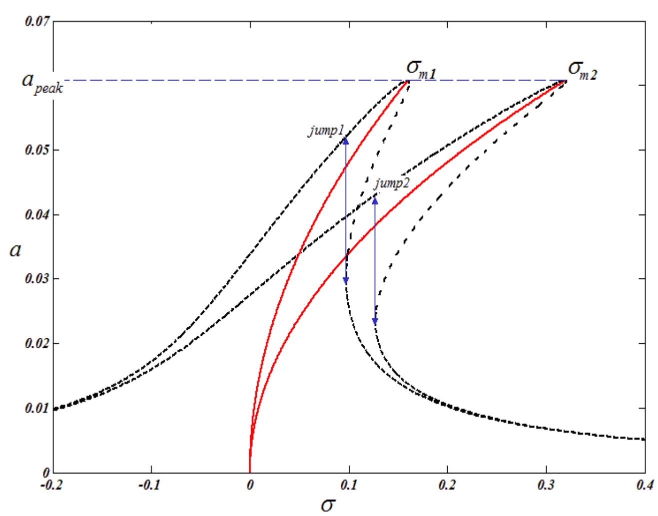

Figure 2: Frequency response of system for two typical cases

The maximum value of amplitude is independent from the nonlinear parameter and obtained by $a_{peak} = q/2\omega\mu$, and the nonlinear term influences on bending the curve. The red lines are known as backbone curves to be achieved by the following equation [16]

$$\sigma = 3\alpha . a^2 / 8\omega \tag{15}$$

APPLYING OPTIMIZATION

- Determining of the Objectives

To define the objective functions for optimization, let consider the case such that σ is approaching zero from right side. As seen in both graphs of Figure 1, a jumping phenomenon, where the amplitude of vibration increases unexpectedly, occurs in the frequency response which is undesirable for the system [16]. It is evident from Figure 2 that for identical a_{peak}, the system with higher curvature, i.e., more nonlinear, has lower undesirable jump. Also, the figure reveals that for similar σ, the system with more inclination to the vertical line reaches both higher amplitude and larger range of jump. On the other hand, for such vibrational system it is desirable to have lower amplitude of vibration when the system is imposed by an external force.

Accordingly, the aim is to avoid larger jump and higher amplitude for the vibration of system; which is achieved by defining a bi-objective problem with following two objectives: maximizing of σ_m and minimizing of a_{peak}. The external force and damping are not the design parameters hence the aim is to design the geometry and location of stiffeners affected by natural frequency, nonlinear coefficient, and subsequently by σ_m and a_{peak}. Eventually, the objective functions of our problem are defined as follows:

$$\begin{cases} Objective\ 1: \min\left\{f_1(h,t,t_1,s) = 1/\sigma_m\right\} \\ Objective\ 2: \min\left\{f_2(h,t,t_1,s) = a_{peak}\right\} \end{cases} \tag{16}$$

- Decision variables

The following design parameters build our decision variables: thickness of the plate (h), thickness of the stiffener (t), height of the stiffener (t1), and average distance of two adjacent stiffeners (s). The last one can be defined by the number of stiffeners N, used at the span of a plate, where $s = a / N$. The variable N is a discrete parameter unlike the others. As the aim is to employ an evolutionary optimization method (i.e., multi-objective genetic algorithm) which can handle mix-type variable problems (continuous and discrete ones) [22]; also non-differentiable functions [24]; however, by defining *s* as a continuous variable and using it instead of *N*, there would be no discrete variable in the decision parameters; which simplifies our optimization approach. There are some box constraints on above decision variables based on the standards of designing stiffened plate; which should be taken into account in optimization [20-21]. Accordingly, the following constraints are defined for the corresponding variables; which are applicable for the type and size of original plate in this work.

$$\begin{cases} h_{\min} \le h \le h_{\max}, \\ 1h \le t \le 5h, \\ 1h \le t_1 \le 2h, \\ \dfrac{a}{10} \le s \le \dfrac{a}{5} \ (5 \le N \le 10 \ per\ span\ of\ the\ plate) \end{cases} \quad (17)$$

- *Optimization method: Genetic Algorithm (GA)*

A multi-objective genetic algorithm is implemented for solving the problem in this paper. This method employs an iterative stochastic search strategy to find Pareto optimal solutions by imitating principles of biological evolution [18]. In GA, also other evolutionary algorithms, a population of individuals is used as potential candidate solutions of optimization problem; where any individual consists of the values of the designing parameters [19]. The utilized GA-based multi-objective optimization algorithm obtains a local Pareto front for multiple objective functions. This approach works on a population of individuals using a set of biological operators (i.e., selection, crossover, and mutation) which are applied on the population in which the initial population is generated randomly, generally speaking [23]. The next generation of the population is computed based on the non-dominated ranking using a distance-based measure of the current individuals in the population [24]. A non-dominated rank is assigned to each individual using the relative fitness values, for example, the individual 'a' dominates individual 'b' if 'a' is strictly better than 'b' in at least one objective and 'a' is no worse than 'b' in all other objectives [24]. In other word, 'b' is dominated by 'a'; or 'a' is non-inferior to 'b'. Two individuals 'a' and 'b' are considered to have equal ranks if neither dominates the other. The distance measure of an individual is used to compare individuals with an equal rank [24]. It is a measure of how far an individual is from the other individuals with the same rank. The algorithm uses a controlled elitist GA favors individuals that can help increase the diversity of the population even if they have a lower fitness value [22, 24]. It is very important to maintain the diversity of population for convergence to an optimal Pareto front which can be done by controlling the elite members of the population as the algorithm progresses [24]. A Pareto fraction option limits the number of individuals on the Pareto front (elite members) and the distance function helps to maintain diversity on a front by favoring individuals that are relatively far away from the front [24]. Further information on the fundamental concepts of a GA is discussed in [19].

RESULTS AND DISCUSSION

In this section, the results of optimization are presented by considering variant values for the length of the right-angle plate, upper and lower bounds of the plate, thickness, and amplitude of excitation. A section of the Pareto Front for considered two-objective functions are presented in Figure 3 for specific numerical values. Here, three important points are chosen and the corresponding frequency responses are plotted in Figure 4. As seen, for point C the curvature of the backbone is the highest one so the system shows it is the most nonlinear behavior. However the amplitude of vibrations reaches the maximum value for this case. On the contrary, point A shows the most linear behavior having smallest amplitude. The point B represents a moderate behavior in terms of nonlinearity and amplitude of vibration which can be an appropriate option for a design problem. In addition, the corresponding values of objective functions and decision variables for abovementioned points have been provided in the Table 1.

Table 1. Some sample Pareto solutions: Objectives and variables for three points *A*, *B*, *C*

Points	A	B	C
f_1	0.736	6.188	49.739
f_2	0.098	0.061	0.043
h	0.015	0.015	0.011
t	0.016	0.027	0.030
t_1	0.067	0.057	0.075
s	0.228	0.191	0.182

Figure 5 shows the variations of both objectives when the right angle side length, *A*, increases. It is obvious by decreasing the length has a positive effect on the optimal solutions; however, the diversity of results is not appropriated. The effect of changing excitation amplitude on objectives is presented in the Figure 6.

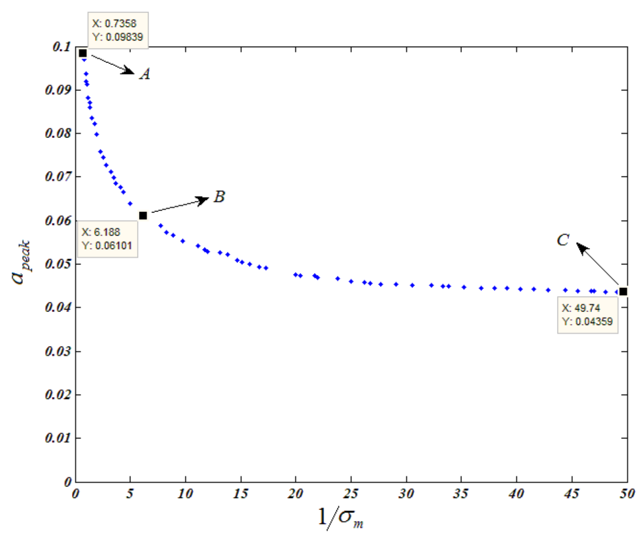

Figure 3: Distribution of Pareto Front (PF) solutions

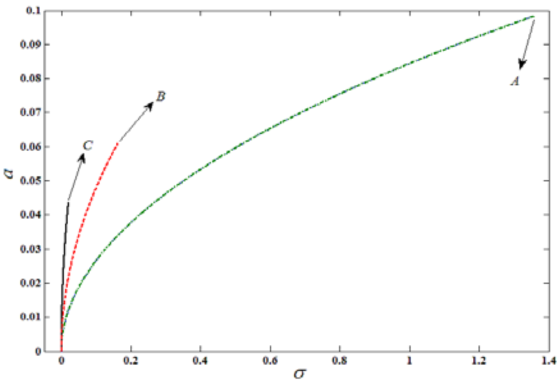

Figure 4: Frequency responses of three special points

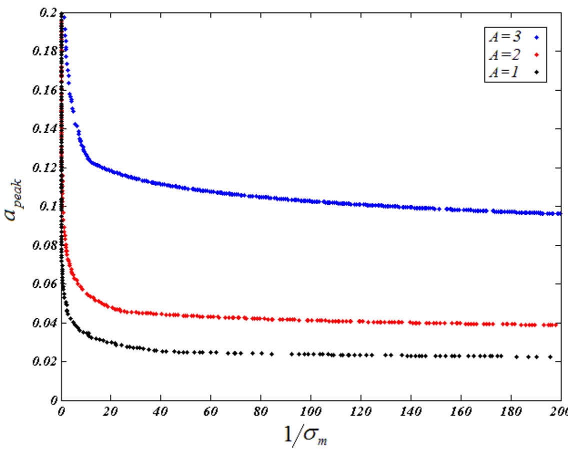

Figure 5: Effects of the plate length *A*, on PF solutions

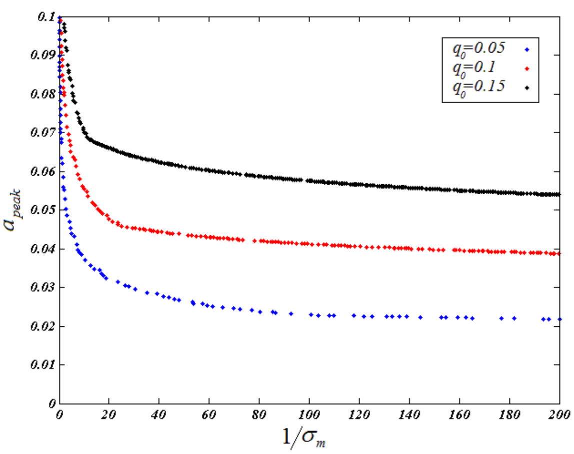

Figure 6: Effects of excitation amplitude on PF solutions

Here, decrease in the excitation amplitude leads to better optimal results, unlike the diversity. Also, variation of upper and lower bounds of plate thickness has been shown in the Figure 7, where by increasing the bounds, the optimal solutions have been improved. The interactions of decision variables are shown in Figure 8. Their shape's complexity is the evidence for the complexity of solved bi-objective problem in term of the variables' interaction.

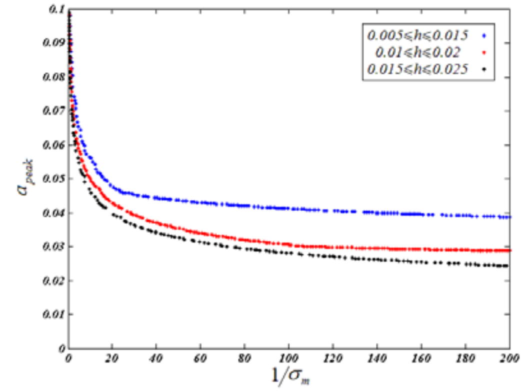

Figure 7: Effects of the thickness boundaries on PF solutions

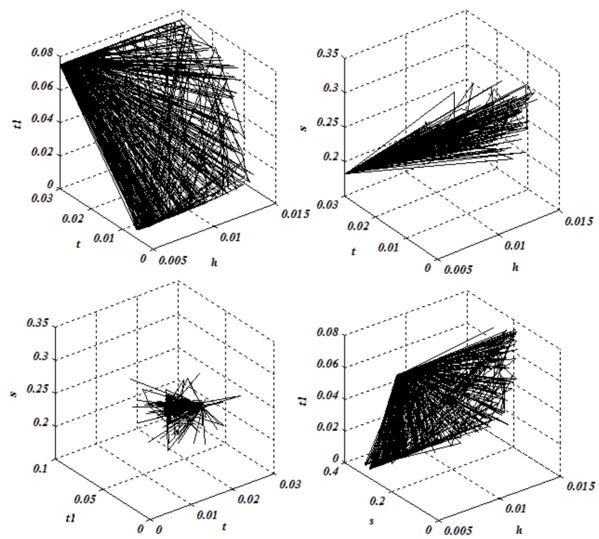

Figure 8: Variables' interaction

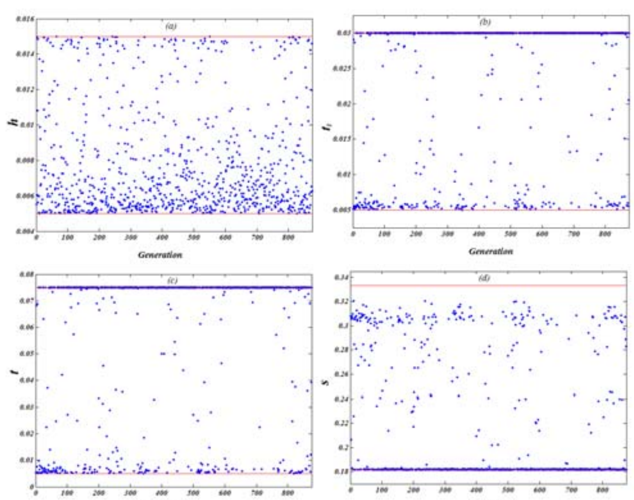

Figure 9: Scattering of variables during generations' progress.

Figure 5 depicts the distribution of decision variables between their upper and lower bounds for a typical case.

As seen, the variables *t* and t_1 (i.e., stiffener thickness and height, respectively) tend to their upper bounds but the parameter *s* has more tendency to its lower bound. In addition, the distribution of the parameter *h* as the plate thickness is almost uniform with a higher tendency toward the lower bound.

CONCLUSION

In this paper, nonlinear force vibration of a triangular plate was studied. The plate includes stiffeners; and the primary resonance was considered in vibration behavior of the plate. The governing equation of motion was derived and solved by means of the Galerkin procedure and the method of multiple scales. Subsequently, the frequency response was presented in an analytical form. In next step, a multi-objective optimization was carried out considering two conflicting objective functions, i.e., maximizing the nonlinearity of the system and minimizing the amplitude of vibration.

Four decision variables were taken into account including the thickness of the plate, the geometry, and the distance of the stiffeners. Results of optimization using a genetic algorithm were reported in detail. Finally, the effects of different parameters on the optimal solutions and also the distribution of decision variables were investigated to provide a comprehensive understanding about the system's behavior.

It was found any decrease on the distance between the two consecutive stiffeners and any increase on the width of the stiffeners will reduce the amplitude of vibration, as well as, the system's nonlinear behavior. Thus, if it is desired for the system to have lower amplitude of vibration with a linear behaviour; it would be necessary to use the highest allowable number of stiffeners with the largest possible width in the span of the plate. Increase on the height of stiffener can also be useful for having this behaviour; however this parameter and also the thickness, *h*, do not always show a regular behavior for their influence on the response of the system. As a future work, one can suggest to take two mode shapes of vibration for the system which results two coupled ordinary differential equations for the time response of the system; and consequently the number of objective functions in the optimization problem increases to four functions.

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